Simplified theory of an active lift turbine (ALWT) with controlled displacement and blade pitch control

Pierre Lecanu[1]

[1] P. Lecanu,(Corresponding author) EXECO, 14000 Caen, France *E-mail address*, normandajc@posteo.net

This document is a rewrite of the April 2015 preprint "Simplified theory of an active lift turbine with controlled"(**lecanu:hal-0130053**). The first version of the active lift turbine with a crankshaft system had radial displacements. The relative speed that creates an induced force on the profile depends on the profile's rotational speed, the fluid's speed, and the radial displacement speed. The latter speed reduces the angle of incidence and therefore reduces the induced force. In the 2015 preprint, there was a version in which the profile's trajectory was close to a circle but which required a complex mechanism. The turbine presented in this document has significant radial displacement. To compensate for the reduction in the angle of incidence, a mechanism has been added to increase the angle of incidence. The design is simple and robust.

Keywords Betz-limit Betz's-law Wind-turbine Tidal-turbine HAWT VAWT ALWT

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1 Nomenclature

This list is an enumeration of the main symbols used in this article.

Symbol	Designation	Unit
R	Radius	m
c	Chord length of blades	m
$\bar{R}$	Mean radius	m
$\dot{R}$	$\frac{dR}{dt}$	$\frac{m}{s}$
r	Dimemensonless radius	$-$
$\dot{r}$	$\frac{dr}{dt}$	$\frac{m}{s}$
ϵ_1	Geometric parameter for radius variation	$-$
ϵ_2	Geometric parameter for the variation of the angle of incidence	$-$
T	Tangential force	N
N	Normal force	N
i	Angle of incidence without blade pitch control	rad
α	Angle of incidence with blade pitch control	rad
V_∞	free stream velocity	$\frac{m}{s}$
V_u	Upstreamwise velocity at the turbine	$\frac{m}{s}$
V_d	Downstreamwise velocity at the turbine	$\frac{m}{s}$
V_e	Streamwise velocity at the turbine	$\frac{m}{s}$
V_w	Streamwise velocity in the far wake	$\frac{m}{s}$
v	Dimemensonless velocity	$-$
W	Velocity relative	$\frac{m}{s}$
w	Dimemensonless velocity relative	$-$
P	Pressure	Pa
P_{atm}	Atmopheric pressure	Pa
λ	Tip speed ratio	$-$
b	Plenitude speed ratio	$-$
C_T	Tangentiel coefficient	$-$
C_N	Normal coefficient	$-$
T	Tangentiel force	N
N	Normal force	N
Cp	Coeffient power	$-$
Cp_T	Tangentiel coeffient power	$-$
Cp_N	Normal coeffient power	$-$
$ALWT$	Active lift wind turbine	$-$

1.1 Patent of the Active Lift Turbine (ALWT) :

New patent filed for the new lift turbine concept. $FR2506916 \quad jun \;\; 2025$

1.2 Geometry and power of the Active Lift Turbine (ALWT) :

The active-lift turbine is a connecting-rod-crank system. Turbine blades have a circular trajectory with radial displacement. The radius varies with the angle of rotation.

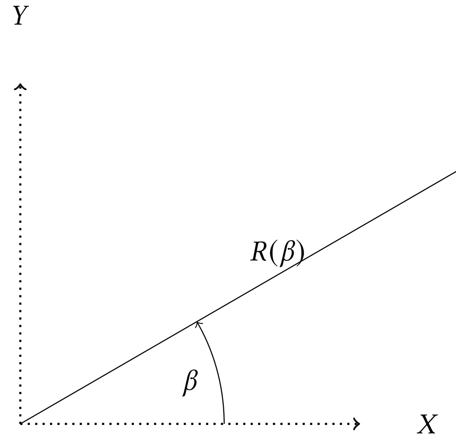


Figure 1: Geometry of ALWT.

$$R : radius \qquad \dot{R} = \frac{\partial R}{\partial t} \qquad \bar{R} : mean\,radius$$

$$R(\beta) \;=\; \bar{R}\;\;(1 + \epsilon_1 \cos(\beta)) \qquad \dot{R} \;=\; -\epsilon_1 \bar{R} \sin(\beta)\;\dot{\beta} \tag{1}$$

ϵ_1 constant positive number
Dimensionless radius

$$r(\beta) \;=\; \frac{R(\beta)}{\bar{R}} \;=\; (1 + \epsilon_1 \cos(\beta)) \qquad \dot{r} \;=\; -\epsilon_1 \sin(\beta)\;\dot{\beta} \tag{2}$$

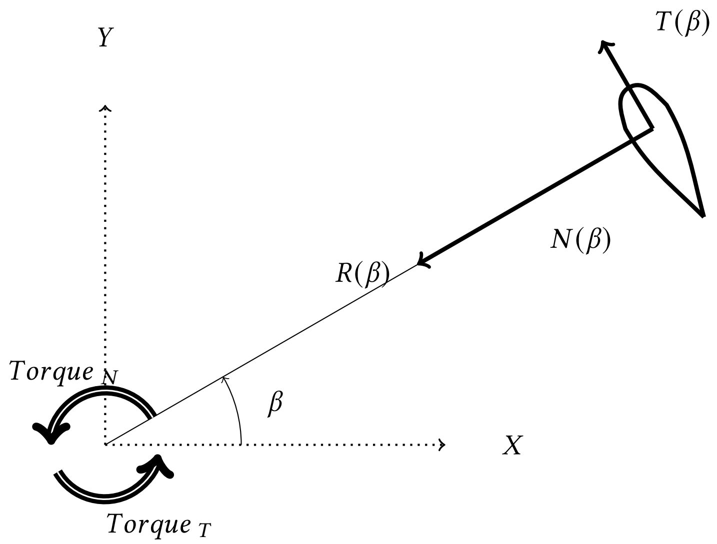


Figure 2: Torques of ALWT

The powers produced by force T and force N are

$$Power_{\,T}\;(\beta) \;=\; T(\beta)\;R(\beta)\;\dot{\beta} \qquad Power_{\,N}\;(\beta) \;=\; N(\beta)\;\dot{R}(\beta) \tag{3}$$

The principle of the creation of the central torque $Torque_{\,N}$ due to the $forceN$ is not described in this article.

For a Darrieus turbine

$$R(\beta) \;=\; \bar{R} \qquad \dot{R}(\beta) \;=\; 0\;\frac{m}{s}$$

$$Power_{\,T}\;(\beta) \;=\; T(\beta)\;\bar{R}\;\dot{\beta} \qquad Power_{\,N}\;(\beta) \;=\; 0\;W$$

1.3 Geometry of blade pitch control (ALWT) :

The speed of radial movement of the blade reduces the angle of incidence. A mechanism not described in this article, allows the profile orientation to be modified in order to increase the angle of incidence.

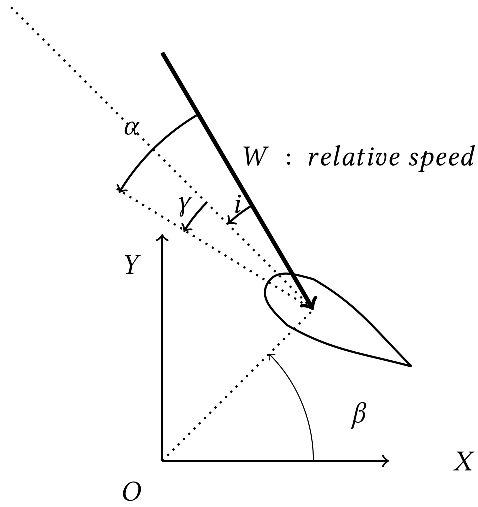


Figure 3: Increased angle of incidence

$$\gamma(\beta) \;=\; \epsilon_2 \;\sin(\beta) \quad with \quad \epsilon_2 \;\; numerical\; constant \tag{4}$$

$$\dot{\gamma}(\beta) \;=\; \epsilon_2 \;\cos(\beta)\dot{\beta} \tag{5}$$

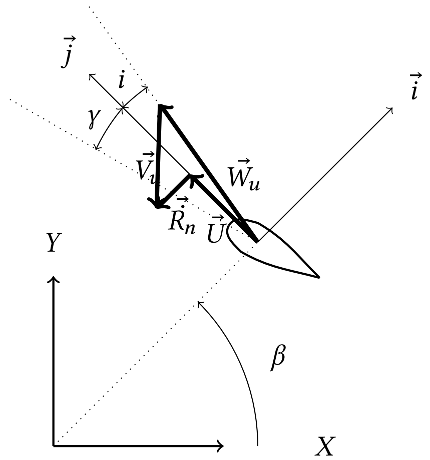


Figure 4: Velocities triangle

velocities triangle

$$\vec{v_u} \;\approx\; \vec{w_u} + \lambda\vec{j} + \dot{r}\vec{i} + \dot{\gamma}\frac{c}{4}\vec{i} \qquad \dot{r} >> \dot{\gamma}\frac{c}{4} \qquad \dot{\gamma}\frac{c}{4} \quad negligible\;\; value$$

The Naca profile follows a deformed circular trajectory during one rotation.

$$v_u \sin\beta \;+\; \dot{r} = w_u \sin i \qquad v_u \cos\beta \;+\; \lambda = w_u \cos i$$

The relative speed can be defined by this approximate formula

$$w(\beta) \;\approx\; \lambda \;+\; v(\beta)\cos\beta \tag{6}$$

By using equations (1),(2), (23),(25) and also the above equations.

$$v_u(\beta) \approx 1 - \frac{b}{4} sin(\beta) \qquad v_d(\beta) \approx 1 - \frac{-3b}{4} sin(\beta) \tag{7}$$

$$v(\beta) \approx \left| \begin{array}{ll} 1 - \frac{b}{4} sin(\beta) & if \quad 0 < \beta \leq \pi \\ 1 - \frac{-3b}{4} sin(\beta) & if \quad \pi < \beta \leq 2\pi \end{array} \right. \tag{8}$$

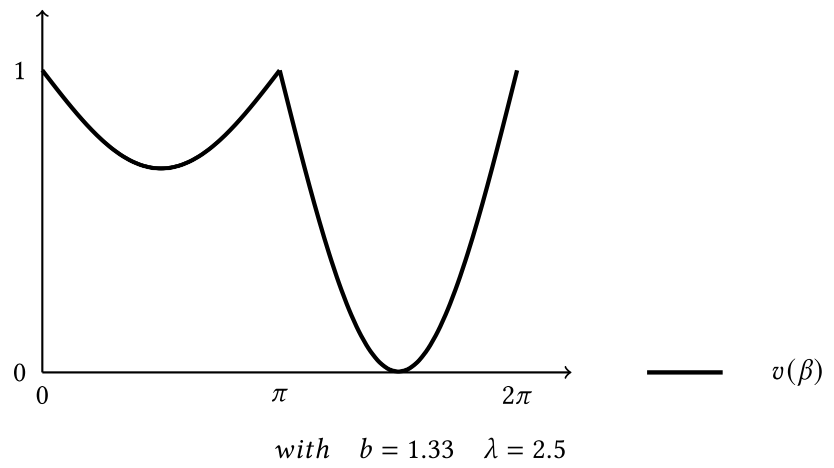


Figure 5: Fluid speed at the turbine

$v(\beta)$ must have a positive value, which imposes $\frac{3b}{4} \leq 1$

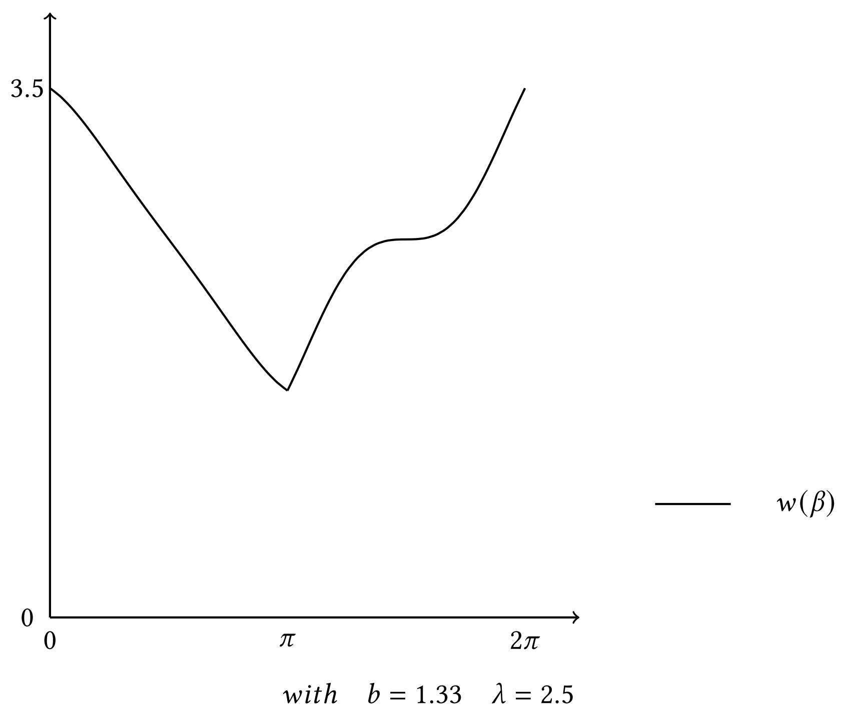


Figure 6: Relative speed at the turbine

Incidence angle The angle of incidence α being small, the cosine of this angle is approximately equal to 1. w can be simplified by this expression By using 9 and 4

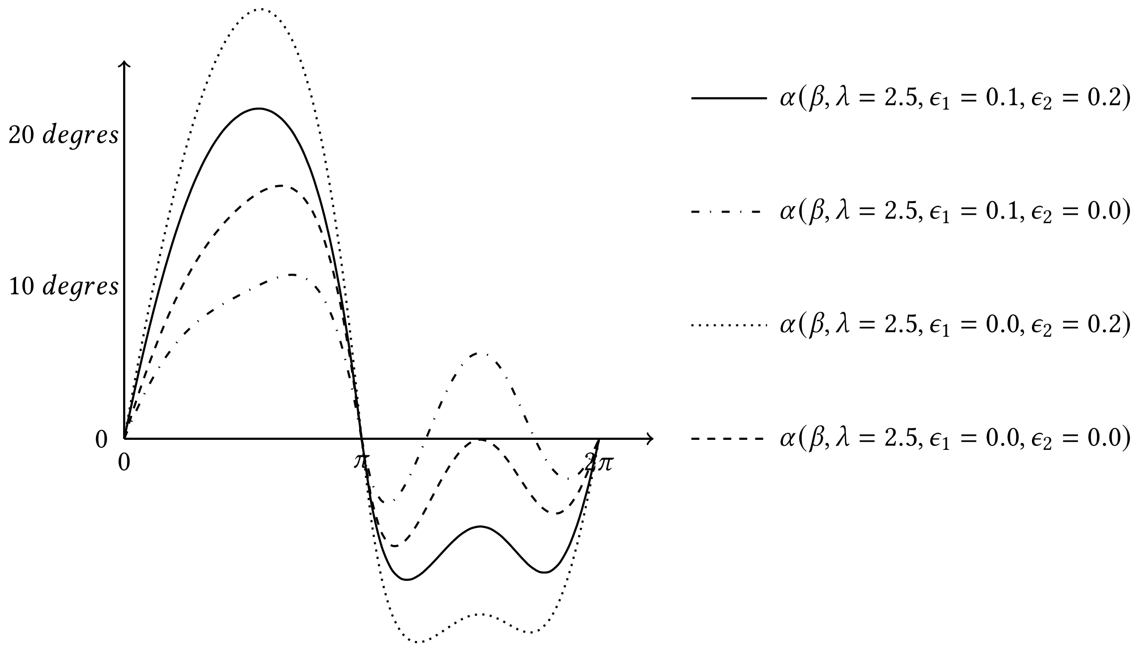
$$\alpha(\beta) = i(\beta) + \gamma(\beta) \qquad \alpha(\beta) \approx \sin(\alpha(\beta))$$

i : angle of incidence without blade pitch control
α : angle of incidence with blade pitch control

$$\alpha(\beta) \approx \frac{v(\beta) sin(\beta) + \dot{r}(\beta)}{w(\beta)} + \gamma(\beta) \tag{9}$$

$$\alpha(\beta, \lambda, \epsilon_1, \epsilon_2) = \frac{\left[v(\beta) - \epsilon_1\lambda + \epsilon_2(\lambda + v(\beta)\cos(\beta))\right]\sin(\beta)}{\lambda + v(\beta)\cos(\beta)} \tag{10}$$

The figure 7 is a graphical representation of the angle of incidence α.



$with \quad b = 1.33$

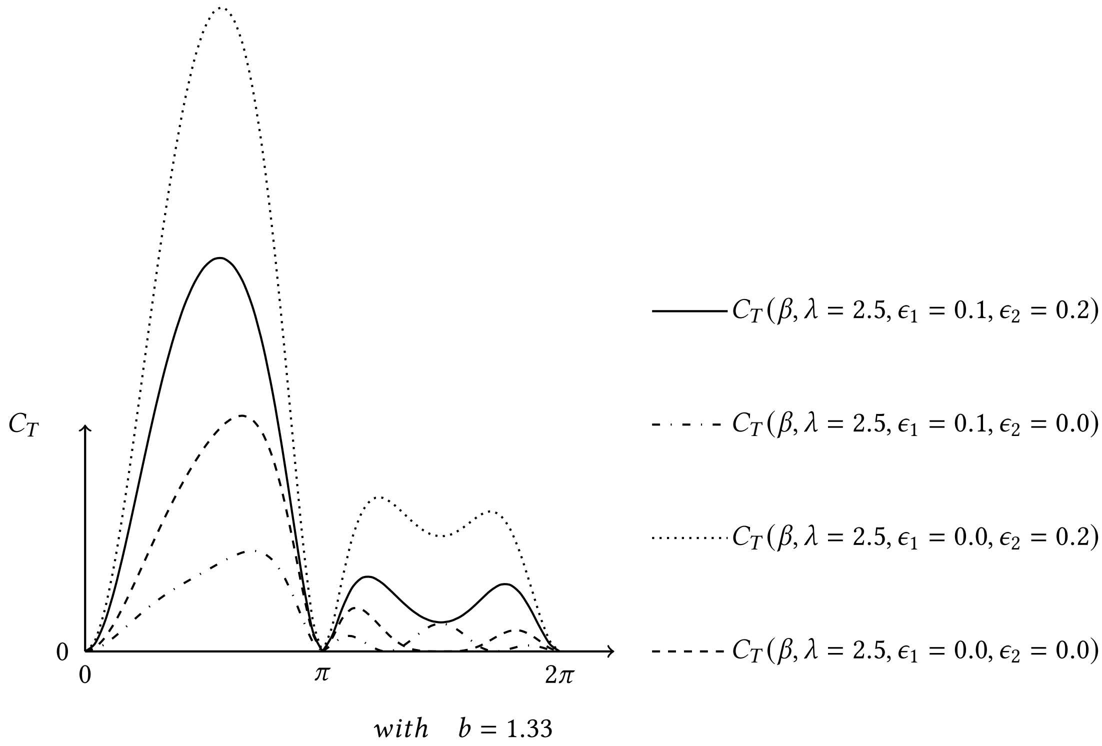
$\epsilon_1 \neq 0 \quad \epsilon_1 \neq 0$ ALWT with blade pitch control
$\epsilon_1 \neq 0 \quad \epsilon_1 = 0$ ALWT without blade pitch control
$\epsilon_1 = 0 \quad \epsilon_1 = 0$ Darrieus turbine

Figure 7: Angle d'incidence α

For a graphical representation, C_N and C_T can be expressed as

$$C_T = 2\pi\,\alpha(\beta)^2 \qquad C_N = -2\pi\,\alpha(\beta) \tag{11}$$

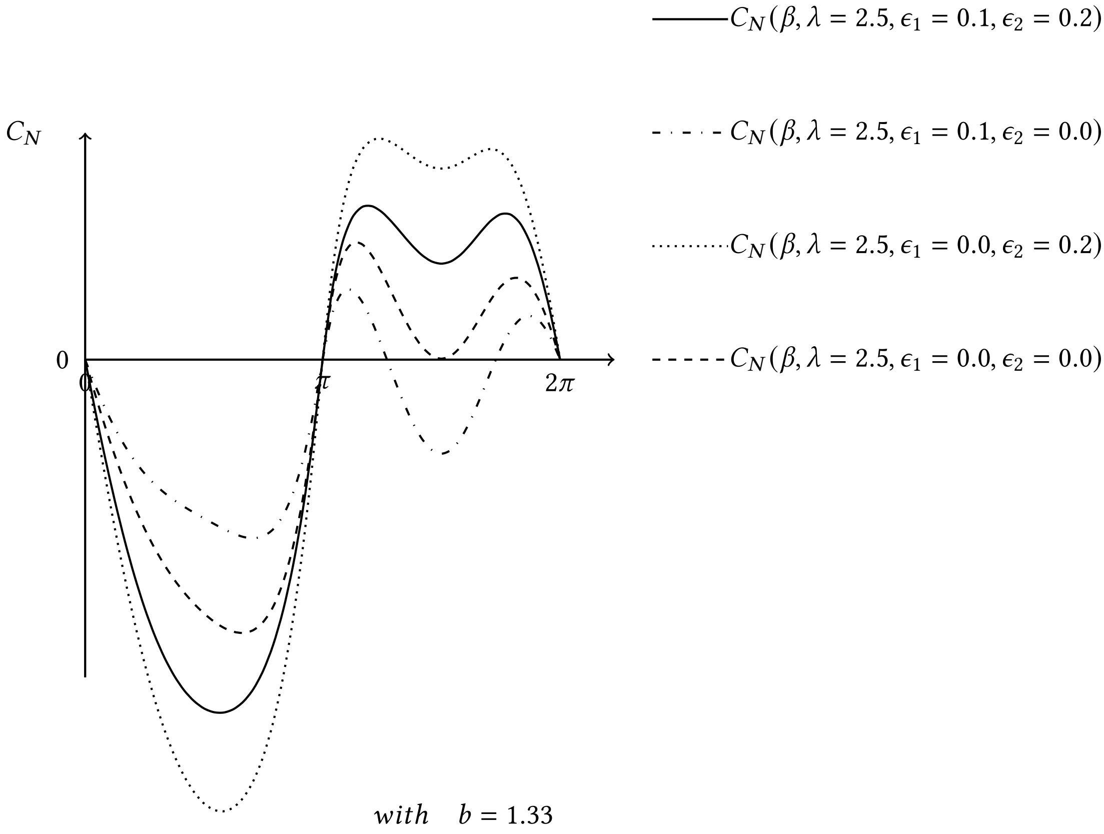
The figure 8 is a graphical representation of the coefficient C_T.



$\epsilon_1 \neq 0 \quad \epsilon_1 \neq 0$ ALWT with blade pitch control
$\epsilon_1 \neq 0 \quad \epsilon_1 = 0$ ALWT without blade pitch control
$\epsilon_1 = 0 \quad \epsilon_1 = 0$ Darrieus turbine

Figure 8: Coefficient C_T

The figure 9 is a graphical representation of the coefficient C_N.



$\epsilon_1 \neq 0 \quad \epsilon_1 \neq 0$ ALWT with blade pitch control

$\epsilon_1 \neq 0 \quad \epsilon_1 = 0$ ALWT without blade pitch control

$\epsilon_1 = 0 \quad \epsilon_1 = 0$ Darrieus turbine

Figure 9: Coefficient C_N

Dimensionless forces are

$$df_T = \frac{dF_T}{F_{wind}} = \frac{b}{2\lambda}\left[v(\beta) - \epsilon_1\lambda + \epsilon_2(\lambda + v(\beta)\cos(\beta))\right]^2 \sin^2(\beta) \quad (12)$$

$$df_N = \frac{dF_T}{F_{wind}} = \frac{-b}{2\lambda}\left[v(\beta) - \epsilon_1\lambda + \epsilon_2(\lambda + v(\beta)\cos(\beta))\right] \sin^2(\beta)\left[\lambda + v(\beta)\cos(\beta)\right] \quad (13)$$

Deflection of the streamtube by referring to paragraph 5.0.0.6

The forces acting on the blades are N and T.(*Concours d'admission 2013 : epreuve de physique filiere PSI* 2013)

$$T = C_T\frac{1}{2}\rho W^2 cdH \qquad N = C_N\frac{1}{2}\rho W^2 cdH \quad dT = \frac{N_p R d\beta}{2\pi\bar{R}}T \quad dN = \frac{N_p R d\beta}{2\pi\bar{R}}N$$

$$W = wV_\infty$$

The wind force is

$$F_{wind} = \frac{1}{2}\rho S_{wept-area}V_\infty^2 = \frac{1}{2}\rho\, 2\bar{R}\, dH\, V_\infty^2$$

Due to the reduction in fluid velocity, there is a deflection of the streamtubes (see figure 10). The deviation of the angle of the streamtube $d\phi$ is of the same order as the variation in angle of $d\beta$. by defining dAx_e as $dAx_e = \frac{dAx_u + dAx_d}{2}$, the normal and tangential forces are

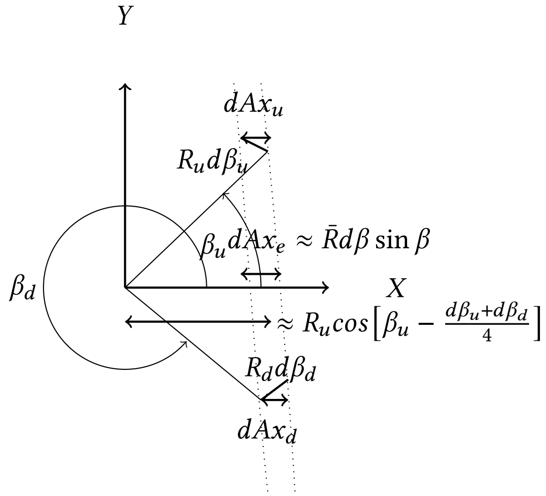


Figure 10: deflection of the streamtube

$$dAx_u\; V_u \;=\; dAx_e\; V_e \;=\; dAx_d\; V_d$$

$$\frac{dAx_u}{dAx_e} = \frac{V_e}{V_u} \qquad \frac{dAx_d}{dAx_e} = \frac{V_e}{V_d} \qquad => \qquad \frac{dAx_e}{dAx_e} = \frac{1}{2}\Big[\frac{V_e}{V_u} + \frac{V_e}{V_d}\Big] \qquad => \qquad V_e = \frac{2V_u V_d}{V_u + V_d}$$

$$\frac{dAx_u}{dAx_e} = \frac{2V_d}{V_u + V_d} \qquad \frac{dAx_d}{dAx_e} = \frac{2V_u}{V_u + V_d}$$

$$R_u d\beta = \frac{dAx_u}{sin\beta} = \frac{1}{sin\beta}\frac{dAx_u}{dAx_e} dAx_e = \frac{2V_d}{V_u + V_d}\bar{R}\, d\beta \qquad R_d d\beta = \frac{2V_u}{V_u + V_d}\bar{R} d\beta$$

coefficient Power calculation by using (3) $dP_{\,T} = dT\; R\; \dot{\beta}$ $\quad dP_{\,N} = dN\; \dot{R}$ the power coefficients are

$$dCP_T = \frac{dP_T}{F_{wind} V_\infty}$$

$$dCP_N = \frac{dP_N}{F_{wind} V_\infty}$$

By using (10),

$$dCP_T u = \frac{dT_u\; R_u\; \dot{\beta}}{\frac{1}{2}\rho\, 2\bar{R}\, dH\, V_\infty^3}$$

$$dCP_T d = \frac{dT_d\; R_d\; \dot{\beta}}{\frac{1}{2}\rho\, 2\bar{R}\, dH\, V_\infty^3}$$

$$dCP_N u = \frac{dNu\; \dot{R}}{\frac{1}{2}\rho\, 2\bar{R}\, dH\, V_\infty^3}$$

$$dCP_N d = \frac{dNd\; \dot{R}}{\frac{1}{2}\rho\, 2\bar{R}\, dH\, V_\infty^3}$$

$$dCP_T u = b\big[v_u - \epsilon_1 \lambda + \epsilon_2(\lambda + v_u \cos(\beta))\big]^2 \sin^2(\beta)\frac{R}{\bar{R}}\frac{v_d}{v_u + v_d}\, d\beta$$

$$dCP_Td \;=\; b\left[v_d - \epsilon_1\lambda + \epsilon_2(\lambda + v_d\cos(\beta))\right]^2 \sin^2(\beta)\frac{R}{\bar{R}}\frac{v_u}{v_u + v_d}\; d\beta$$

$$dCP_Nu \;=\; -\epsilon_1\; b\left[v_u - \epsilon_1\lambda + \epsilon_2(\lambda + v_u\cos(\beta))\right](\lambda + v_u cos(\beta))\sin^2(\beta)\frac{v_d}{v_u + v_d}\; d\beta$$

$$dCP_Nd \;=\; -\epsilon_1\; b\left[v_d - \epsilon_1\lambda + \epsilon_2(\lambda + v_d\cos(\beta))\right](\lambda + v_d cos(\beta))\sin^2(\beta)\frac{v_u}{v_u + v_d}\; d\beta$$

$$dCP_u \;=\; dCP_Tu + dCP_Nu \qquad dCP_d \;=\; dCP_Td + dCP_Nd$$

The coefficient power is

$$Cp \;=\; Cp_T \;+\; Cp_N \qquad with$$

$$Cp_T \;=\; \int_0^{\pi} dCP_Tu \;+\; \int_{\pi}^{2\pi} dCP_Td$$

$$Cp_N \;=\; \int_0^{\pi} dCP_Nu \;+\; \int_{\pi}^{2\pi} dCP_Nd$$

$$\begin{aligned} Cp_T \;=\; &\int_0^{\pi} b\left[(1+\frac{-b}{4}\sin\beta - \epsilon_1\lambda + \epsilon_2(\lambda + (1+\frac{-b}{4}\sin\beta)\cos\beta)\right]^2 \sin^2\beta \frac{1+\frac{-3b}{4}\sin\beta}{1+\frac{-b}{4}\sin\beta + 1 + \frac{-3b}{4}\sin\beta} d\beta + \\ &\int_{\pi}^{2\pi} b\left[(1+\frac{-3b}{4}\sin\beta - \epsilon_1\lambda + \epsilon_2(\lambda + (1+\frac{-3b}{4}\sin\beta)\cos\beta)\right]^2 \sin^2\beta \frac{1+\frac{-b}{4}\sin\beta}{1+\frac{-b}{4}\sin\beta + 1 + \frac{-3b}{4}\sin\beta} d\beta \end{aligned} \tag{14}$$

$$\begin{aligned} Cp_N = &\int_0^{\pi} b\epsilon_1\left[(1+\frac{-b}{4}\sin\beta - \epsilon_1\lambda + \epsilon_2(\lambda + (1+\frac{-b}{4}\sin\beta)\cos\beta)\right]\left[\lambda + (1+\frac{-b}{4}\sin\beta)\cos\beta\right]\sin^2\beta \frac{1+\frac{-3b}{4}\sin\beta}{1+\frac{-b}{4}\sin\beta + 1 + \frac{-3b}{4}\sin\beta} d\beta + \\ &\int_{\pi}^{2\pi} b\epsilon_1\left[(1+\frac{-3b}{4}\sin\beta - \epsilon_1\lambda + \epsilon_2(\lambda + (1+\frac{-3b}{4}\sin\beta)\cos\beta\right]\sin^2\beta \frac{1+\frac{-3b}{4}\sin\beta}{1+\frac{-b}{4}\sin\beta + 1 + \frac{-3b}{4}\sin\beta} d\beta \end{aligned} \tag{15}$$

For a Darrieus Turbine ($\epsilon_1 = 0 \quad \epsilon_1 = 0$)

$$Cp_{Darrieus} = \int_0^{\pi} b(1+\frac{-b}{4}\sin\beta)(\frac{-3b}{4}\sin\beta)\sin^2\beta\, d\beta \;=\; b(\frac{9\pi b^2}{128} - \frac{4b}{3} + \frac{\pi}{2})$$

Using Mathcad 15 software to calculate equations 14 and 15, the following results are obtained: (figures 12 and 12)

$$Cp_{Tu}(\varepsilon_1,\varepsilon_2,\lambda,b) := \int_0^{\pi} b\cdot\left[\left[1+\frac{-b}{4}\cdot\sin(\beta)-\varepsilon_1\cdot\lambda+\varepsilon_2\cdot\left(1+\frac{-b}{4}\cdot\sin(\beta)+\lambda\cdot\cos(\beta)\right)\right]^2\cdot\sin(\beta)^2\right]\cdot\left[\frac{1+\frac{-3\cdot b}{4}\cdot\sin(\beta)}{1+\frac{-b}{4}\cdot\sin(\beta)+\left(1+\frac{-3\cdot b}{4}\cdot\sin(\beta)\right)}\right]d\beta$$

$$Cp_{Td}(\varepsilon_1,\varepsilon_2,\lambda,b) := \int_{\pi}^{2\cdot\pi} b\cdot\left[\left[1+\frac{3\cdot b}{4}\cdot\sin(\beta)-\varepsilon_1\cdot\lambda+\varepsilon_2\cdot\left(1+\frac{3\cdot b}{4}\cdot\sin(\beta)+\lambda\cdot\cos(\beta)\right)\right]^2\cdot\sin(\beta)^2\right]\cdot\left[\frac{1+\frac{b}{4}\cdot\sin(\beta)}{1+\frac{b}{4}\cdot\sin(\beta)+\left(1+\frac{3\cdot b}{4}\cdot\sin(\beta)\right)}\right]d\beta$$

$$Cp_T(\varepsilon_1,\varepsilon_2,\lambda,b) := Cp_{Tu}(\varepsilon_1,\varepsilon_2,\lambda,b)+Cp_{Td}(\varepsilon_1,\varepsilon_2,\lambda,b)$$

$$Cp_{Nu}(\varepsilon_1,\varepsilon_2,\lambda,b) := \int_0^{\pi} b\cdot\varepsilon_1\cdot\left[\left[1+\frac{-b}{4}\cdot\sin(\beta)-\varepsilon_1\cdot\lambda+\varepsilon_2\cdot\lambda+\left(1+\frac{-b}{4}\cdot\sin(\beta)\right)\cdot\cos(\beta)\right]\cdot\left[\lambda+\left(1+\frac{-b}{4}\cdot\sin(\beta)\right)\cdot\cos(\beta)\right]\right]\cdot\sin(\beta)^2\cdot\frac{\left(1+\frac{-3\cdot b}{4}\cdot\sin(\beta)\right)}{\left(1+\frac{-b}{4}\cdot\sin(\beta)\right)+\left(1+\frac{-3\cdot b}{4}\cdot\sin(\beta)\right)}d\beta$$

$$Cp_{Nd}(\varepsilon_1,\varepsilon_2,\lambda,b) := \int_{\pi}^{2\cdot\pi} b\cdot\varepsilon_1\cdot\left[\left[1+\frac{3\cdot b}{4}\cdot\sin(\beta)-\varepsilon_1\cdot\lambda+\varepsilon_2\cdot\lambda+\left(1+\frac{3\cdot b}{4}\cdot\sin(\beta)\right)\cdot\cos(\beta)\right]\cdot\left[\lambda+\left(1+\frac{3\cdot b}{4}\cdot\sin(\beta)\right)\cdot\cos(\beta)\right]\right]\cdot\sin(\beta)^2\cdot\frac{1+\frac{b}{4}\cdot\sin(\beta)}{1+\frac{b}{4}\cdot\sin(\beta)+\left(1+\frac{3\cdot b}{4}\cdot\sin(\beta)\right)}d\beta$$

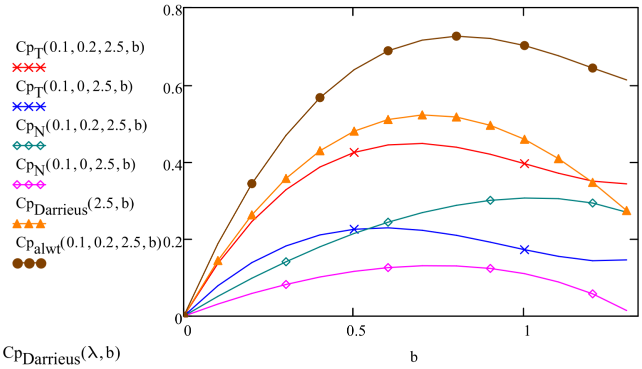
$$Cp_N(\varepsilon_1,\varepsilon_2,\lambda,b) := Cp_{Nu}(\varepsilon_1,\varepsilon_2,\lambda,b)+Cp_{Nd}(\varepsilon_1,\varepsilon_2,\lambda,b)$$

$$Cp_{Darrieus}(\lambda,b) := Cp_T(0,0,\lambda,b)$$

$$Cp_{alwt}(\varepsilon_1,\varepsilon_2,\lambda,b) := Cp_T(\varepsilon_1,\varepsilon_2,\lambda,b)+Cp_N(\varepsilon_1,\varepsilon_2,\lambda,b)$$

Figure 11: Using Mathcad 15 software, calculate the power coefficient using equations 14 and 15

$Cp_{Darrieus}(2.5,1) = 0.458$ $\qquad b := 0, 0.1 .. \frac{4}{3}$



$$gain_{Darrieus}(\varepsilon_1,\varepsilon_2,\lambda,b) := \frac{Cp_{alwt}(\varepsilon_1,\varepsilon_2,\lambda,b)-Cp_{Darrieus}(\lambda,b)}{Cp_{Darrieus}(\lambda,b)}$$

$gain_{Darrieus}(0.1,0.1,1.5,1) = 4.02\cdot\%$	$Cp_{alwt}(0.1,0.1,1.5,1) = 47.678\,\%$	$gain_{Darrieus}(0.1,0.2,1.5,1) = 35.485\cdot\%$	$Cp_{alwt}(0.1,0.2,1.5,1) = 62.101\,\%$
$gain_{Darrieus}(0.1,0.1,2.5,1) = 1.041\cdot\%$	$Cp_{alwt}(0.1,0.1,2.5,1) = 46.313\,\%$	$gain_{Darrieus}(0.1,0.2,2.5,1) = 53.143\cdot\%$	$Cp_{alwt}(0.1,0.2,2.5,1) = 70.194\,\%$
$gain_{Darrieus}(0.1,0.1,3.5,1) = 6.629\cdot\%$	$Cp_{alwt}(0.1,0.1,3.5,1) = 48.874\,\%$	$gain_{Darrieus}(0.1,0.2,3.5,1) = 91.363\cdot\%$	$Cp_{alwt}(0.1,0.2,3.5,1) = 87.712\,\%$

Figure 12: Using Mathcad 15 software, ploted and gain obtained from the equations of figure 11

An approximation of the power coefficient calculation can be used to give a rough idea.

$$Cp_{alwt\ approx} = (1+\epsilon_1\lambda)b\left(\frac{9\pi b^2}{128}-\frac{4b}{3}+\frac{\pi}{2}\right)$$

remark

Since the Reynolds number is sufficiently high ($R_e = 1.7\ 10^6$) the theoretical part considers the fluid to be an perfect fluid. At the blade surfaces, this is not true and there is vortex shedding. The theory does not take these phenomena into account. The geometric coefficient $epsilon_2$ chosen is 0.2. A lower $epsilon_2$ coefficient would increase the tangential force and reduce the normal force. Optimization work needs to be carried out. The aim of this article is to prove that it is possible to effectively improve the efficiency of a wind turbine.

2 Conclusion

Controlled angle of incidence allows curves similar to those of a Darrieus turbine to be obtained. Without angle of incidence control, the results are disastrous. This is why the active lift turbine with an off-center axis was developed. This version of the turbine is mechanically complex to manufacture. The active lift turbine with controlled angle of incidence controled exceptional results (see figure 12) with simple and robust mechanics. The study was conducted with a low lambda value. A conventional offshore wind turbine with a $\lambda > 4$, for example, 2 MW could produce more than 3 MW with an ALWT (with blade pitch control) turbine. It's a huge power gain

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3 Theory on vibrations :

3.1 Resonant modes

The relative velocity is not uniform during each turn. This generates velocity variations of the rotational velocity and induced vibrations. By using Lagrange equations (virtual powers), we will estimate these vibrations

The velocity variations are

$\dot{\beta}$ has been considered as constant $\dot{\beta} = \omega = constant$

The rotational angular velocity ω follows the variations.

In order to take into account these variations, the angular velocity can be defined as

$$\dot{\beta} = \dot{\bar{\beta}} + \dot{\acute{\beta}} \qquad \dot{\bar{\beta}} = \omega = Mean\,constant\,velocity$$

The velocity variations compared to the mean velocity are corresponding to the terms $\dot{\acute{\beta}}$

First approximation is giving $\dot{\acute{\beta}}$ by $a\,\ddot{\beta}$ a being a constant

$$\dot{\beta} = \dot{\bar{\beta}} + a\,\ddot{\beta} = \omega + a\,\ddot{\beta}$$

The kinetic energy is

$$T_e = \frac{1}{2} m\{(R\dot{\beta})^2 + \dot{R}^2\} = \frac{1}{2} m\bar{R}^2\dot{\beta}^2\{1 + 2\epsilon_1 \cos\beta + \epsilon_2^2\}$$

Considering the profil mass being in one point m for simplify calculations

The Lagrange equations terms at the kinetic energy level are

$$\frac{\partial T_e}{\partial \dot{\beta}} = m\bar{R}^2\dot{\beta}\{1 + 2\epsilon_1 \cos\beta + \epsilon_2^2\}$$

$$\frac{d}{dt}\{\frac{\partial T_e}{\partial \dot{\beta}}\} = m\bar{R}^2\ddot{\beta}\{1 + 2\epsilon_1 \cos\beta + \epsilon_2^2\} - 2\epsilon_1 m\bar{R}^2\dot{\beta}^2 \sin\beta$$

$$\frac{\partial T_e}{\partial \beta} = -\epsilon_1 m\bar{R}^2\dot{\beta}^2 \sin\beta$$

$$\frac{d}{dt}\{\frac{\partial T_e}{\partial \dot{\beta}}\} - \frac{\partial T_e}{\partial \beta} = m\bar{R}^2\ddot{\beta}\{1 + 2\epsilon_1 \cos\beta + \epsilon_2^2\} - \epsilon_1 m\bar{R}^2\dot{\beta}^2 \sin\beta$$

4 Powers calculation

By using

$$F_w = \frac{1}{2}\rho 2\bar{R}dHV_\infty^2$$

and using this approximate dimensionless force df_T(12) by

$$df_T(\beta, k) = \frac{1}{N_p}\frac{-b}{6.5}\frac{1}{\lambda}\left[\sin^2(\beta + k\frac{2\pi}{N_p}) + \frac{1}{2}\sin(\beta + k\frac{2\pi}{N_p})\right] \quad with \quad 0 \le k < N_p$$

and using this approximate dimensionless force df_N(**??**) by

$$df_N(\beta, k) = \frac{1}{N_p}\frac{-b}{1.9}\frac{1}{\sqrt{\lambda}}\left[\sin(\beta + k\frac{2\pi}{N_p}) + \frac{1}{2}\sin^2(\beta + k\frac{2\pi}{N_p})\right] \quad with \quad 0 \le k < N_p$$

the force T by blade is $T(k) = df_T(\beta, k)F_w$ and the force N by blade is $N(k) = df_N(\beta, k)F_w$

The figure 13 is a graphical representation of the comparison of theoretical and approximate dimensionless force df_T. The figure 14 is a graphical representation of the comparison of theoretical and approximate dimensionless force df_N.. The figure 15 is a graphical representation of approximate dimensionless force $df_T(\beta, k)$ with different values of k. The figure 16 is a graphical representation of approximate dimensionless force $df_N(\beta, k)$ with different values of k.

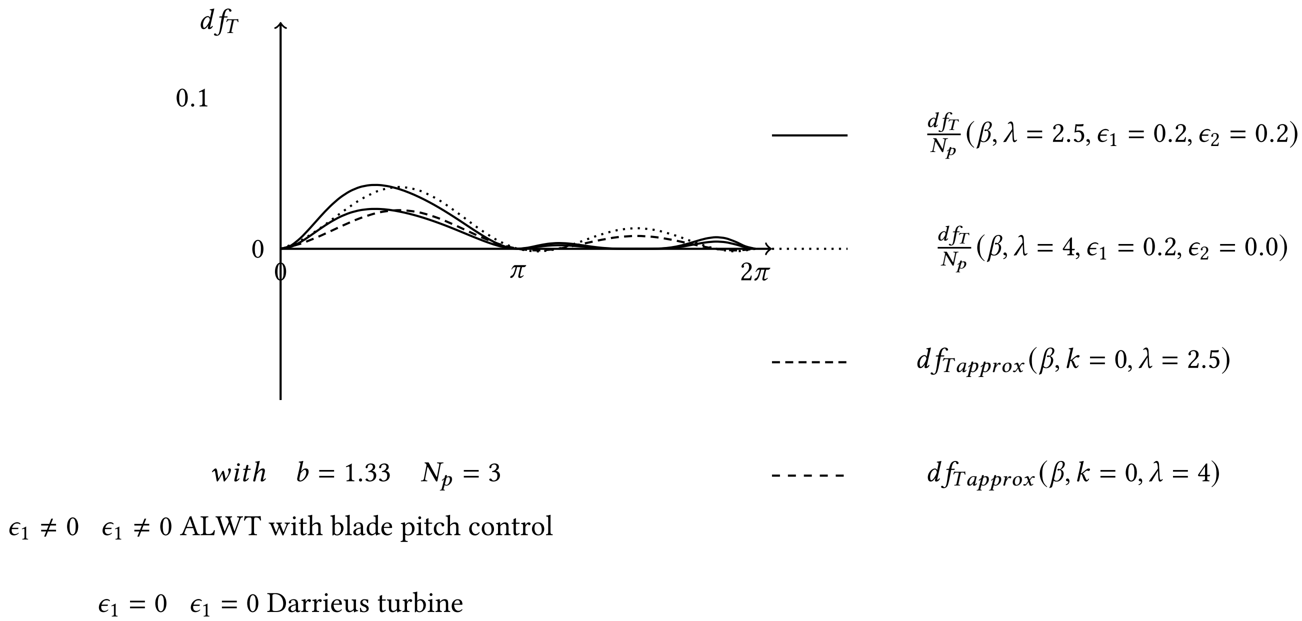

Figure 13: comparison of theoretical and approximate dimensionless force df_T

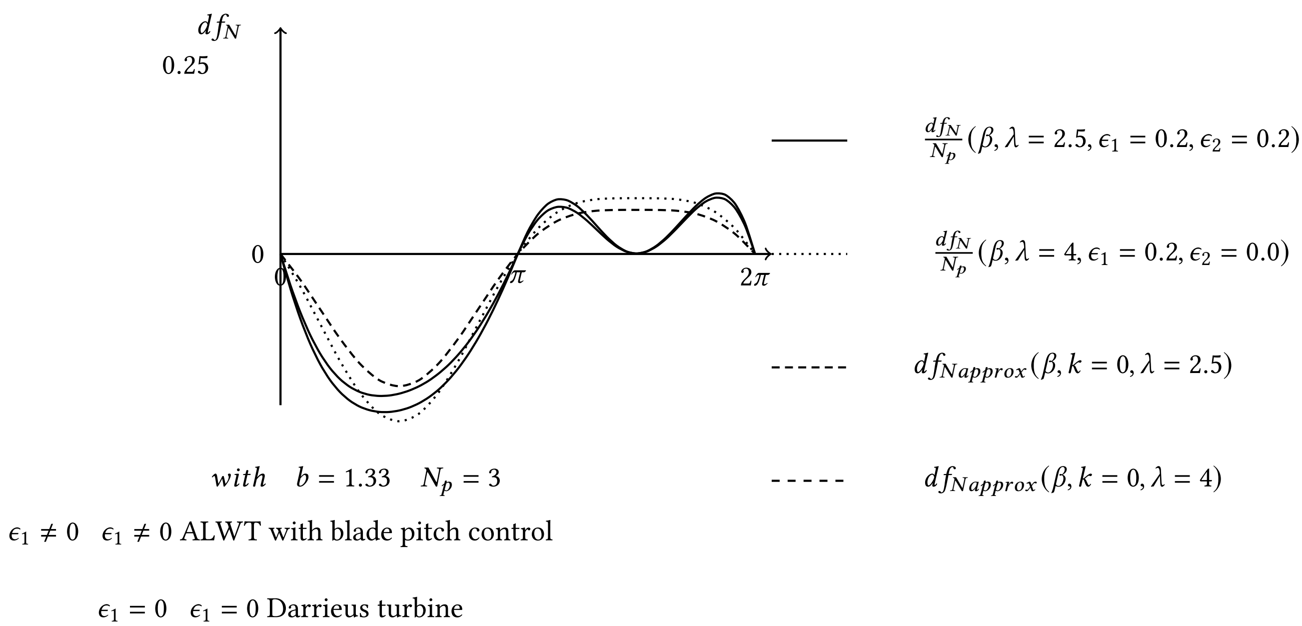

Figure 14: Comparison of theoretical and approximate dimensionless force df_N.

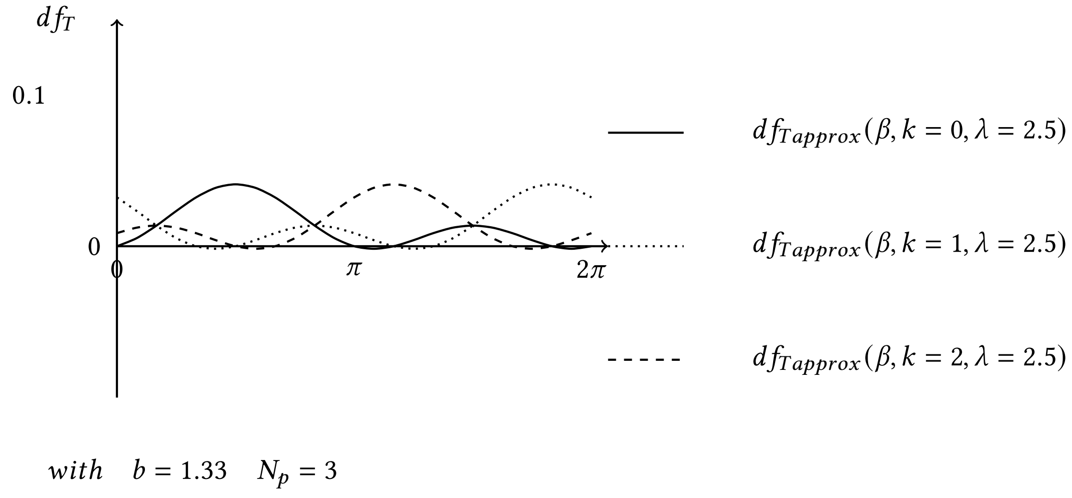

Figure 15: Approximate dimensionless force $df_T(\beta, k)$ with different values of k

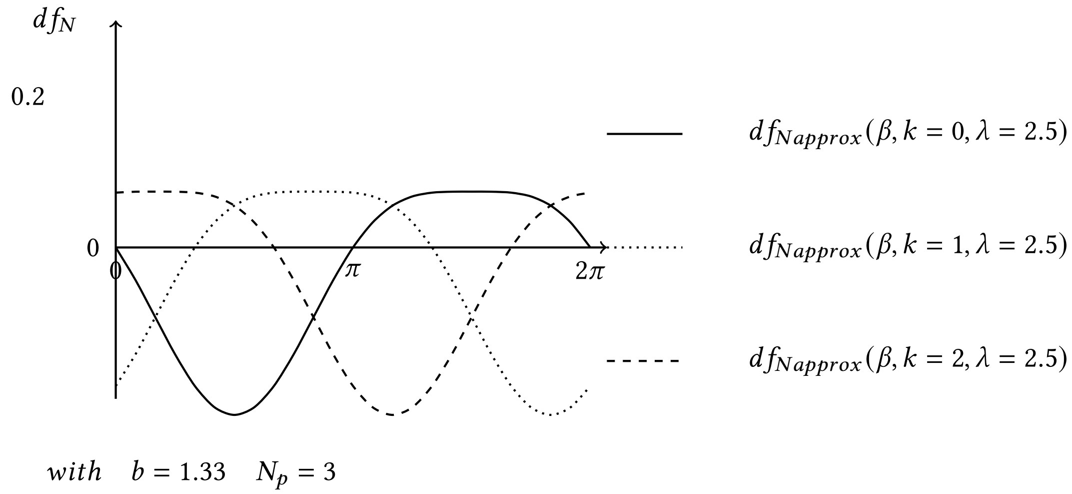

Figure 16: Approximate dimensionless force $df_N(\beta, k)$ with different values of k

$$Power_{Driving} = \{N\vec{\imath} + T\vec{\jmath}\} \wedge \{\dot{R}\vec{\imath} + TR\dot{\beta}\vec{\jmath}\} = N\dot{R} + TR\dot{\beta}$$

$$N\dot{R} + TR\dot{\beta} = N(-\epsilon_1 \bar{R}\sin(\beta)\dot{\beta}) + TR\dot{\beta} = (-\epsilon_1 N\bar{R}\sin(\beta) + TR\dot{\beta})\dot{\beta}$$

$$Power_{Resisting} = -C_{Resisting}\,\dot{\beta}$$

$$C_{Resisting} \;:\; resisting\, torque\, due\, electrical\, generator\,, to\, the\, friction,\; etc$$

The lagrange equation terms of the power level are

$$A_\beta = \left[-\epsilon_1 N\bar{R}\sin(\beta) + TR\dot{\beta}\right]$$

$$A_\beta = \frac{F_w \bar{R} b}{N_p} * \sum_{k=0}^{k=N_p-1} \frac{1}{6.5\lambda}\Big[\sin^2(\beta + k\frac{2\pi}{N_p}) + \frac{1}{2}\sin(\beta + k\frac{2\pi}{N_p})\Big] + \frac{\epsilon_1}{6.5\lambda}\Big[\sin^2(\beta + k\frac{2\pi}{N_p}) + \frac{1}{2}\sin(\beta + k\frac{2\pi}{N_p})\cos(\beta + k\frac{2\pi}{N_p})\Big] + \frac{\epsilon_1}{1.9\sqrt{\lambda}}\Big[\sin^2(\beta + k\frac{2\pi}{N_p}) + \frac{1}{2}\sin^3(\beta + k\frac{2\pi}{N_p})\Big] \tag{16}$$

The Lagrange equation is

$$\frac{d}{dt}\{\frac{\partial T_e}{\partial \dot{\beta}}\} - \frac{\partial T_e}{\partial \beta} = A_\beta$$

The Lagrange equation for a Darrieus turbine type ($\epsilon_1 = 0 \quad \epsilon_2 = 0$)

$$\ddot{\beta} = \frac{F_w b}{m\bar{R}N_p}\sum_{k=0}^{k=N_p-1}\frac{1}{6.5\lambda}\Big[\sin^2(\beta + k\frac{2\pi}{N_p}) + \frac{1}{2}\sin(\beta + k\frac{2\pi}{N_p})\Big] - C_{resisting}$$

5 Darrieus turbine :

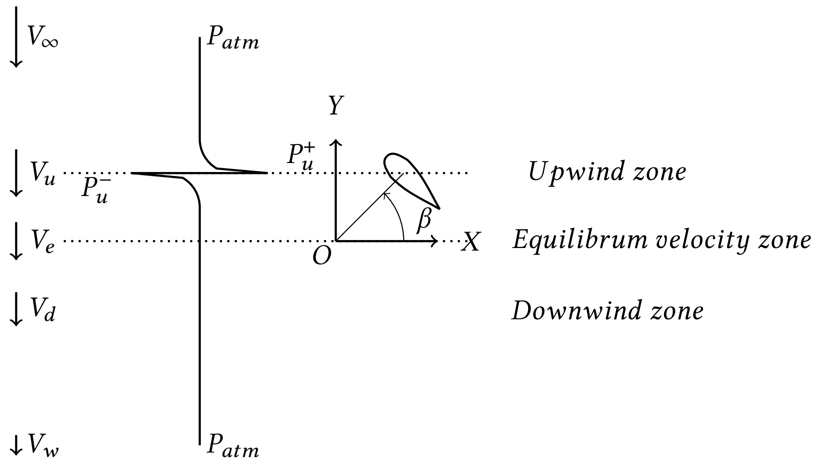

Figure 17: Velocity and pressure

5.0.0.1 The Bernoulli equations

$$\vec{V_\infty} = V_\infty \vec{Y} \qquad with \qquad V_\infty \leq 0$$

$$P_{atm} + \frac{1}{2}\rho V_\infty^2 = P_{u+} + \frac{1}{2}\rho V_u^2$$

$$P_{u-} + \frac{1}{2}\rho V_u^2 = P_{atm} + \frac{1}{2}\rho V_e^2 = P_{d+} + \frac{1}{2}\rho V_d^2$$

$$P_{d-} + \frac{1}{2}\rho V_d^2 = P_{atm} + \frac{1}{2}\rho V_w^2$$

From these equations, we can defined these pressure differences

$$P_{u+} - P_{u-} = \frac{1}{2}\rho(V_\infty^2 - V_e^2) \qquad P_{d+} - P_{d-} = \frac{1}{2}\rho(V_e^2 - V_w^2) \tag{17}$$

5.0.0.2 The continuity equation along a streamtube

$$dAx_u V_u = dAx_d V_d \quad with \quad dAx_u = R_u d\beta_u \sin\beta_u dH \quad dAx_d = R_d d\beta_d \sin\beta_d dH$$

$$dH : elementary\, height\, of\, the\, blade$$

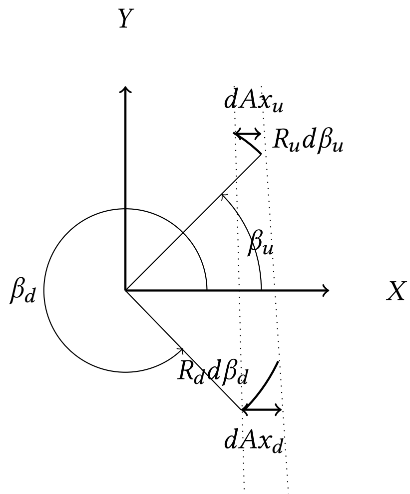

Figure 18: Streamtube

We can define the relation between these two elementary surfaces

$$dAx_d = dAx_u \frac{V_u}{V_d} \tag{18}$$

5.0.0.3 Forces on a blade crossing a streamtube

$$0 \leq \beta \leq \pi \quad dFy_u = (P_{u+} - P_{u-})dAx_u \quad dFx_u = (P_{u+} - P_{u-})dAx_u\frac{1}{tan\beta}$$

$$\pi \leq \beta \leq 2\pi \quad dFy_d = (P_{d+} - P_{d-})dAx_u\frac{V_u}{V_d} \quad dFx_d = (P_{d+} - P_{d-})dAx_u\frac{V_u}{V_d}\frac{1}{tan\beta}$$

By using(18), these equations become

$$dFy_u = \frac{1}{2}\rho(V_\infty^2 - V_e^2)dAx_u \quad dFy_d = \frac{1}{2}\rho(V_e^2 - V_w^2)dAx_u\frac{V_u}{V_d} \tag{19}$$

$$dFx_u = \frac{1}{2}\rho(V_\infty^2 - V_e^2)dAx_u\frac{1}{tan\beta} \quad dFx_d = \frac{1}{2}\rho(V_e^2 - V_w^2)dAx_u\frac{V_u}{V_d}\frac{1}{tan\beta} \tag{20}$$

5.0.0.4 Euler Equations

$$dFy_u = (V_\infty - V_e)dm_u \quad dm_u = \rho V_u dAx_u$$

$$dFy_d = (V_e - V_w)dm_d \quad dm_d = \rho V_d dAx_d \tag{21}$$

By using(19) and the previous equations, we determine the following relations between the velocities

$$V_e = 2V_u - V_\infty \qquad V_w = 2(V_d - V_u) + V_\infty$$

Finally, the Euler equations give

$$dFy_u = 2(V_\infty - V_u)\rho V_u dA_u \quad dFy_d = 2(2V_u - V_d - V_\infty)\rho V_u dA_u \tag{22}$$

5.0.0.5 Projection of the aerodynamic force Induced velocity create an aerodynamic force on the profile which can be decomposed into a lift and a drag force.

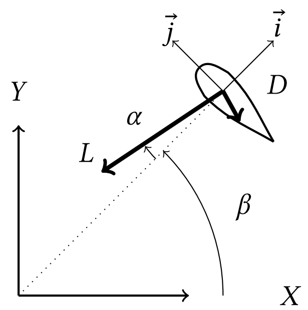

Figure 19: Lift and drag force and the blade

At the turbine level, it is important to know the decomposition of this force into a normal force N, and a tangential force T.

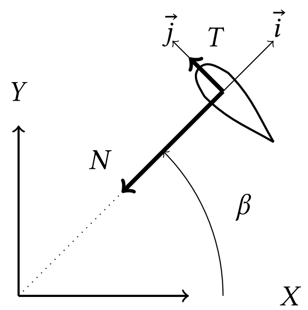

Figure 20: Normal and tangential force and the blade

Forces are set according to X and Y axis from the normal force N and the tangential force T.

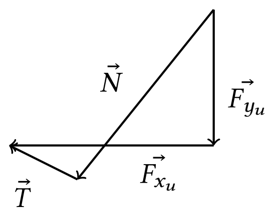

Figure 21: Resolution of forces

$$Fy_u = N\sin\beta - T\cos\beta \qquad Fx_u = N\cos\beta + T\sin\beta$$

$$T = L\sin\alpha - D\cos\alpha \qquad N = L\cos\alpha + D\sin\alpha$$
$$L = C_L\frac{1}{2}\rho W^2 cdH \qquad D = C_D\frac{1}{2}\rho W^2 cdH \qquad c : blade\, chord$$
$$T = C_T\frac{1}{2}\rho W^2 cdH \qquad N = C_N\frac{1}{2}\rho W^2 cdH$$

Thanks to mathematical approximation, drag and lift coefficients can be defined for a symmetrical profile of a Naca0012 type and for low incidence values.

$$C_L = 2\pi\sin\alpha \qquad C_D = constant$$

$$C_D\, has\, a\, value\, far\, below\, C_L\, in\, the\, order\, of\, C_D \; < \; 10\, C_L \qquad C_D \; \approx \; 0$$

$$C_T = 2\pi\sin^2\alpha - C_D \approx 2\pi\sin^2\alpha \qquad C_N = \; -2\pi\sin\alpha\cos\alpha \tag{23}$$

$$Fy_u = \frac{1}{2}\rho W_u^2 cdH\sin\beta(C_N - C_T\frac{\cos\beta}{\sin\beta})$$
$$and \quad Fy_d = \frac{1}{2}\rho W_d^2 cdH\sin\beta(C_N - C_T\frac{\cos\beta}{\sin\beta}) \tag{24}$$

5.0.0.6 Calculation of the value of Fy_u For a two blades turbine, only one blade follows a half-turn.(Reddy 1976)

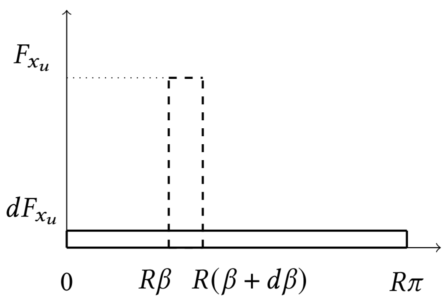

Figure 22: Equality of surfaces

The following equation is given for a turbine with Np blades and by integration :

$$dFy_u R\pi = Fy_u Rd\beta\frac{N_p}{2} \; Fy_u = dFy_u\frac{2\pi}{N_p d\beta} \qquad Fy_d = dFy_d\frac{2\pi}{N_p d\beta}$$

By using the equations (22) and (24) and previous equations

$$\frac{1}{8\pi}\frac{N_p c}{R}W_u^2(C_N - C_T\frac{\cos\beta}{\sin\beta}) = (V_\infty - V_u)V_u$$
$$\frac{1}{8\pi}\frac{N_p c}{R}W_d^2(C_N - C_T\frac{\cos\beta}{\sin\beta}) = (2V_u - V_d - V_\infty)V_u \quad (25)$$

using velocities dimensionless, the equations become

$$v_u = \frac{V_u}{V_\infty} \qquad w_u = \frac{W_u}{V_\infty} \qquad \dot{r} = \frac{\dot{R}}{V_\infty}$$

$$\frac{1}{8\pi}\frac{b}{\lambda}w_u^2(C_N - C_T\frac{\cos\beta}{\sin\beta}) = (1 - v_u)v_u$$
$$\frac{1}{8\pi}\frac{b}{\lambda}w_d^2(C_N - C_T\frac{\cos\beta}{\sin\beta}) = (2v_u - v_d - 1)v_u \quad (26)$$

with

$$\lambda : velocity\ coefficient \qquad \lambda = \frac{\bar{R}\dot{\beta}}{V_\infty}$$

$$b : Plenitude\,speed\,ratio \qquad b = \frac{N_p c\lambda}{\bar{R}}$$